MATHEMATICAL MODELLING Series 1

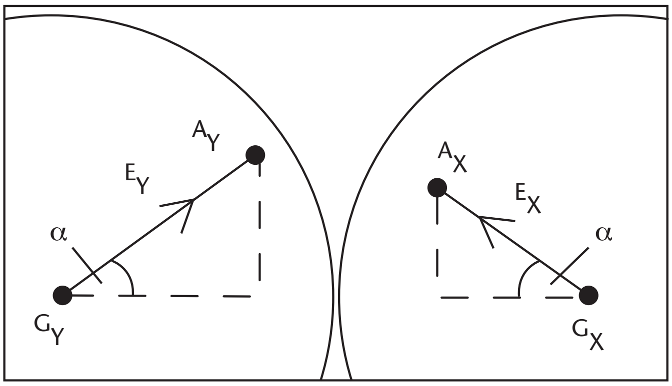
MACSI

STEPS ENGINEERS IRELAND

Top Gear

Mathematics and applied mathematics are used in everyday life. Stock markets, mobile phones, car manufacturing, Google, Hollywood special effects, digital TV and satellites all use cutting-edge mathematics tools in their basic functions. The Mathematical Modelling Series presents a number of applications of mathematics in domains as varied as the human body, volcanology, telecommunications or finance.

Have you ever wondered why some cars make a horrible noise when driving in reverse? Or why offshore wind turbines are so expensive to maintain? Unwanted, undesirable noise, vibration or wear can be a problem in any geared system. Adding oil to the gears can help to reduce the problem, but this is not a practical solution, as it increases the power consumption and thus the cost. Can mathematicians find a cheaper solution?



How it works

When a pair of identical gears, such as those found in a car gear box, are in contact, they operate quietly. However, most materials expand as they get hot. Gear pairs are therefore designed to fit together loosely, so that they will not jam if the teeth become warm. As a result teeth may lose and re-establish contact, with an audible impact. Repeated impacts are known as rattle.

Ideally, gears rotate around their geometric centres (G_X, G_Y) see diagram above; however, in practice the axes of rotation (A_X, A_Y) are not exactly in the centres of the gears (see exaggerated diagram). This difference results in an oscillatory force, which causes teeth to lose and re-establish contact. Engineers can take measurements to calculate an 'eccentricity vector' (E_X, E_Y) for each gear. We employ trigonometry to show that mounting gears 'long axes together' reduces the amplitude of the forcing effect due to eccentricity (E). We use Newton's second law to write down an ordinary differential equation that describes the relative motion of both gears. When the gears are in contact this equation can be solved exactly. The solution can then be used to calculate a maximum value of E, above which quiet operation is impossible.

Conclusion

We can calculate a value for the maximum eccentricity, which is a few tens of microns. Gears with similar eccentricities can be matched to ensure that their combined eccentricity is smaller than this value, and thus quiet operation is possible. Mathematics has found a cheaper alternative to adding more oil to rattling gears!

Parts of the curriculum used in this project:

- Trigonometry
- Differentiation
- Integration
- Newton's second law
- Collisions
- Ordinary differential equations

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If you want more information about MACSI and this project:

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