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MATHEMATICS WORKSHEETS FOR SCHOOLS
Volume 1: Number 3

a. Curriculum Links:
   i. Solving Systems of Equations
   ii. Conservation of Energy
b. Levels:
   i. Junior Certificate Mathematics
   ii. Junior Certificate Science

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Maths makes the perfect shower

Heating and cooling are two activities that contribute to our comfort or discomfort. Whether it is trying to stay warm in winter or cool in summer we never seem to be totally happy with the ambient temperature. In this challenge sheet we will look at an example of a mixing problem that will exhibit conservation of energy[1] and see how Algebra leads to a solution. While mechanical engineering can require some complex mathematics, fortunately a large range of problems can be solved using simple applications of fundamental mathematics. The example we are going to look at here involves the mixing of water at two different temperatures, commonly known as your morning shower. The mechanical engineers who deal with this type of problem are often called Building Services Engineers.

Mathematical Information

We will need to use Algebra in this problem to solve two equations such as:

$$x + y = 5$$

$$2x + 3y = 12$$

A particularly simple way of solving this pair of equations is

1. Use the first equation to find x in terms of y;
2. Substitute this value of x into the second equation;
3. Solve this new equation for y;
4. Substitute this value of y into the first equation to find x.

[1]*What is the conservation of energy? Energy cannot be created or destroyed. In the mixing problem there will be a transfer of energy (heat in this example) from a hot substance to a cold substance.*

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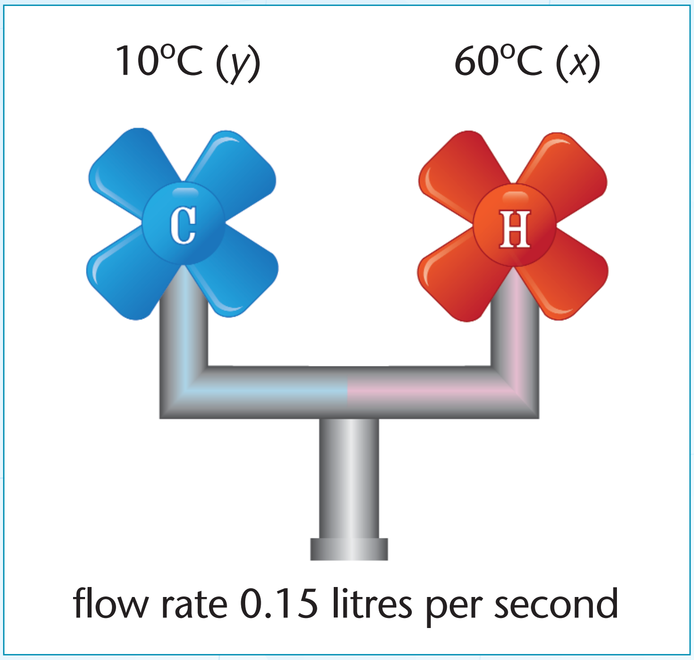


Example[2]

In the majority of houses, water is delivered to the bathroom sink and shower from two separate sources. One of these is a boiler that normally heats water to about 60°C and the other is the cold water which, in Ireland, normally has a temperature of about 10°C. The temperature of 60°C is much too hot for contact with human skin. Cold water must be added to it to bring the overall temperature down to a more comfortable 40°C. Cold water gains heat from the hot water which results in the temperature of the cold water increasing and the hot water decreasing. We will calculate how much cold water is needed to give a final temperature of 40°C. If x litres of hot water is added to y litres of cold water then the total amount of water is $x + y$. In a shower the flow rate is about 0.15 litres per second. This means that every second 0.15 litres of water is delivered to the shower so

$$x + y = 0.15$$

This gives us one equation which is not sufficient on its own to help us calculate the amounts of hot and cold water to mix. We want it so that when we mix x litres of hot water at 60°C with y litres of cold water at 10°C we get 0.15 litres of water at 40°C. It is known from experiments that if you mix 1 litre of water at 60°C with 1 litre of water at 10°C you will get two litres of water whose temperature is between 60°C and 10°C. The temperature of the hot water will decrease as thermal energy is transferred to the cold water and as a result the temperature of the cold water will increase until both litres of water are at the same temperature.

In effect, the heat lost by the hot water will equal the heat gained by the cold water. The heat gained or lost by the water is proportional to the temperature change of the water and the amount (or litres) of water. Since heat is gained or lost by the same substance i.e. water, and the heat lost by the hot water equals the heat gained by the cold water for each second, we can write:

$$\Delta T_h \, x = \Delta T_c \, y$$

$$(60 - 40)x = (40 - 10)y$$

or

$$20x = 30y$$

where ΔT_h (= $T_h - T_f$ = 60 – 40) is the change in the temperature of the hot water and ΔT_c (= $T_f - T_c$ = 40 – 10) is the change in the temperature of the cold water. In these equations, T_h is the temperature of the hot water, T_c is the temperature of the cold water and T_f is the final temperature of the shower.

Step 1:

$$x + y = 0.15$$

$$x = 0.15 - y$$

Step 2:

$$20(0.15 - y) = 30y$$

Step 3:

$$3 - 20y = 30y$$

$$-50y = -3$$

$$50y = 3$$

$$y = \frac{3}{50} = 0.06$$

Step 4:

$$x = 0.15 - 0.06 = 0.09$$

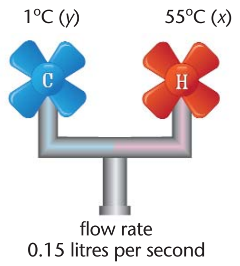
Therefore 0.09 litres per second of water at 60°C and 0.06 litres of water at 10°C will give 0.15 litres of water at 40°C.

[2] *This example is a valid engineering application based on a small number of important scientific principles e.g. conservation of energy. Deeper treatment of this problem will be carried out in the next activity sheet.*

Challenges

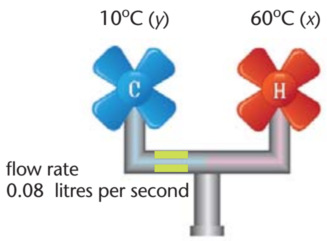
CHALLENGE 1

During the bitterly cold winter of 2010/11 the hot water had cooled to a temperature of 55°C before it reached the shower. The cold water coming into the shower had a temperature of 1°C. The flow rate of the shower is still 0.15 litres per second. Calculate the flow rates of the hot and cold water that will allow you a comfortable shower at 40°C?



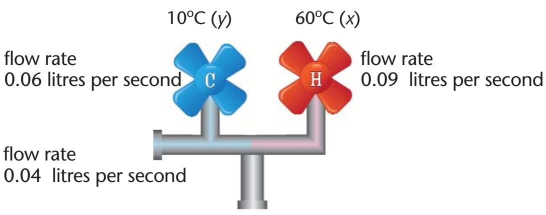
CHALLENGE 2

Cold water, with a temperature of 10C, flows into the shower at a flow rate of 0.08 litres per second. The hot water is still coming in at a temperature of 60C. The desired temperature of the shower is 40C.
How much hot water must flow in per second and what is the total amount of water arriving each second?



CHALLENGE 3

Now assume that you are having a shower at a comfortable temperature of 40°C with the hot (60°C) water flowing at 0.09 litres per second and the cold (10°C) water flowing at 0.06 litres per second.
Someone else in the house turns on a cold tap and takes out cold water at the rate of 0.04 litres per second from the system. What temperature do you now feel in the shower?



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Teacher Page

CHALLENGE 1:

This is a simple variation on the example which can be solved by letting x litres be the amount of hot water arriving per second and y litres the amount of cold water flowing per second.

$$\begin{aligned} x + y &= 0.15 \\ \Rightarrow x &= 0.15 - y \\ 15x &= 39y \\ \Rightarrow 15(0.15 - y) &= 39y \\ 2.25 - 15y &= 39y \\ \Rightarrow y &= 0.042 \\ \Rightarrow x &= 0.108 \end{aligned}$$

which means that 0.042 litres of cold water and 0.108 litres of hot water arrive in the shower each second.

CHALLENGE 2:

Let x litres be the amount of hot water flowing per second and y litres the total amount of water entering the shower every second.

$$\begin{aligned} (60 - 40)x &= (40 - 10) \text{ X } 0.08 \\ \Rightarrow 20x &= 30(0.08) \\ \Rightarrow 20x &= 2.4 \\ \Rightarrow x &= 0.12 \\ y &= x + 0.08 \\ \Rightarrow y &= 0.2 \end{aligned}$$

which means that 0.2 litres of water in total are delivered to the shower each second. Of this 0.12 litres are hot.

CHALLENGE 3:

This is an example of using the equations we have already derived. Since 0.04 litres of cold water is being removed every second only 0.02 litres per second reaches the shower. The flow of water through the shower is now 0.11 litres per second. Let x be the new temperature of the shower. Remember that the heat lost by the hot water equals the heat gained by the cold water and from this we have:

$$\begin{aligned} (60 - x) \text{ X } 0.09 &= (x - 10) \text{ X } 0.02 \\ 60 \text{ X } 0.09 + 10 \text{ X } 0.02 &= 0.11 \text{ X } x \\ \Rightarrow 5.6 &= 0.11x \\ \Rightarrow x &= \frac{5.6}{0.11} \\ \Rightarrow x &= 50.9 \approx 51^{\circ}\text{C} \end{aligned}$$

which is distinctly uncomfortable!

Comments and Suggestions

Overview: Students will manipulate and solve simultaneous equations and then relate their answers back to the contextual problem.

Hints: If you have solved simultaneous equations with your students by equating coefficients there is no need to change your methods. Any method can be used.

You can get many more practical problems like these by considering the calculation of the density of two fluids, specific heat capacity of a mixture, conservation of energy etc.

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