MACSI

STEPS ENGINEERS IRELAND

Maths camouflage

Picture produced by the Plasma Research Laboratory, DCU, Ireland.

Mathematics and applied mathematics are used in everyday life. Stock markets, mobile phones, car manufacturing, Google, Hollywood special effects, digital TV and satellites all use cutting-edge mathematics tools in their basic functions. The Mathematical Modelling Series presents a number of applications of mathematics in domains as varied as the human body, volcanology, telecommunications or finance.

Can mathematics prevent hot spots?

Plasma is a gas in which all the electrons have been stripped from the atoms. On Earth you can find it in fluorescent lamps and neon signs. Plasmas are also widely used in industrial applications, such as reactive etching in the fabrication of electronic integrated circuits. Industrial plasmas are prone to instabilities, which happen when a normally homogeneous plasma (such as you see inside a fluorescent lamp) suddenly breaks up into alternating bright and dark regions. This can be very dangerous, as the bright regions reach high temperatures (hence 'hot spots'), which could burn and destroy surfaces. Mathematics can be used to understand the formation of this bright-dark pattern.

How it works

The behaviour of the plasma can be captured by systems of differential equations, which describe physical laws such as the movement of electrons and the conservation of energy. Mathematical modelling shows that plasma exhibits a so-called pattern formation behaviour – in this case the emergence of a periodic temperature modulation – which gives rise to hot and cold spots! Spontaneous pattern formation is a spectacular phenomenon, which can be witnessed in many areas, for example in biology. Mathematicians have long used differential equations to explain the natural occurrence of

animal coat patterns, be it the spots on this leopard or the stripes on a zebra. Animals use their spots and stripes to camouflage themselves from predators. Have you ever noticed that some spotty animals can have stripey tails (look at a leopard) but nobody has ever seen a stripey animal with a spotty tail? You can even prove this mathematically!

Conclusion

The hot spots in plasma can be characterised mathematically as the maximum points of the temperature function, which sometimes can develop wide oscillations instead of remaining uniform. Using a combination of mathematical techniques from pattern formation theory and computer simulations, it can be determined which system parameters are more likely to lead to such oscillations, so that they can be avoided in physics experiments and technological settings.

Parts of the curriculum used in this project

- Differentiation
- Energy conservation
- Differential equations
- Trigonometry
- Asymptotics
- Integration
- Geometry
- Fluid mechanics
- Linear equations

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