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MATHEMATICS WORKSHEETS FOR SCHOOLS
Volume 1: Number 6

a. Curriculum Links:
   i. Calculation of percentages
   ii. Index manipulation
   iii. Oil Reservoirs
b. Levels:
   i. Junior Certificate Mathematics
   ii. Junior Certificate Science

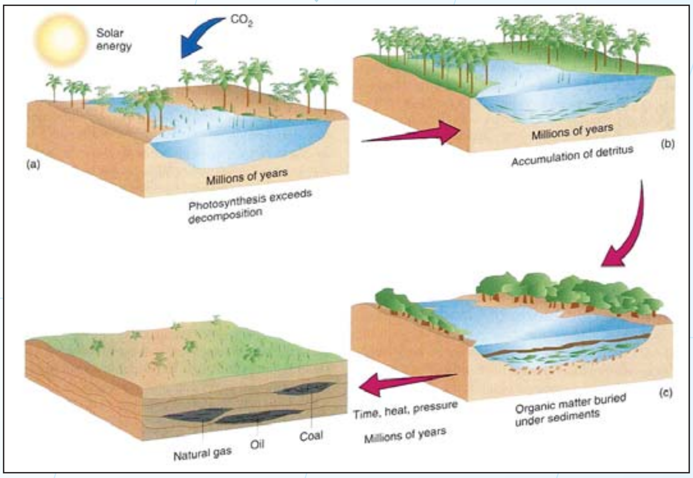
Shell is a global group of energy and petrochemicals companies with around 93,000 employees in more than 90 countries and territories. Its innovative approach ensures it is ready to help tackle the challenges of the new energy future.

Maths strikes oil!

Oil Reservoirs

A reservoir (in terms of oil) is a natural location where hydrocarbons are stored. These locations are typically found up to 3.5 km below the ocean or land surface. However on occasion some reservoirs have been located up to 9 km below the ocean or land surface. Hydrocarbons are chemical substances that consist entirely of hydrogen (H) and carbon (C). Liquid hydrocarbons are referred to as petroleum or mineral oil and gaseous hydrocarbons are referred to as natural gas. In the reservoir, the hydrocarbons tend to be trapped over *porous* rock but under *non-porous* rock. A *porous* rock in this context means rock that permits hydrocarbons to flow through it. *Non-porous* rock does not permit the movement of hydrocarbons.

Reservoirs tend to be located in either sandstone or any carbonate rock which is sealed from above. These types of rock have pore spaces or gaps where oil and gas resides. In effect, any oil reservoir must consist of rock that has *porosity*. *Porosity* is the percentage of the total rock volume that contains space where oil or gas can reside. To explain *porosity*, imagine if you filled a bucket with marbles so that the bucket was filled to the brim. Since the contacting marbles cannot occupy the entire volume of the bucket, there is empty space between the contacts known as pore space or pore volume. For example, you could pour water into the bucket which would fill the pore volume between the marbles. Therefore, the percentage of the volume in the bucket taken up by the water is the *porosity*. However, not all of the pore volume will necessarily be filled with oil or gas. The percentage of the pore volume in the rock that is actually filled with oil is referred to as the *saturation* of the rock.



It is also very important that the sandstone or carbonate rock also have *permeability*. This is a measure of how well fluids can flow through the pore spaces. The larger the permeability of the oil through the rock reservoir, the easier it will be to collect the oil or gas from the reservoir. Finally, there is one last term that is important when talking about oil reservoirs. The *net to gross* is the percentage of the rock volume that is of a high enough quality to be considered reservoir rock. It is usually expressed as a decimal so that a net to gross of 87% would normally be represented as 0.87. This means that if the total volume of rock is 100 m^3 only 87 m^3 of the rock contains oil.

Uncertainty

In an experiment where making measurements of a quantity is important, there will be an associated uncertainty with the measurement of this quantity. The source of these uncertainties can sometimes be due to human error, but invariably, the most important source of these uncertainties is the finite accuracy of measuring instruments. When the quantity we are interested in estimating is not directly visible (such as an estimate of the amount of oil in the reservoir), the uncertainty in the quantity is dependent on the uncertainty in other measurements e.g. *area, height, porosity*, that are used to calculate the quantity. All the ingredients that make up a reservoir provide uncertainties for the geoscientist. A geoscientist performs a test drill to get an estimate of the rock and oil conditions for a particular location. Using the test drill, geoscientists take the core or rock samples from the reservoir.

With these samples, they can estimate the *porosity*, *permeability, saturation* and *net to gross* of the reservoir at that location. A number of such test drills will be performed to give the geoscientist an idea of the rock conditions throughout the reservoir. A drill may inform a geoscientist of the rock conditions at the drill site but there is no way of knowing if these conditions persist 1 km away, which is why a number of test drills are performed at different sites.

When estimating the amount of oil present in a reservoir the first step is to assume that the oil occupies a shape that can be approximated as a regular solid. The volume of such a shape is

$$\text{volume} = \text{area} \times \text{height}$$

and both the area and the height (or depth of oil) are estimated from drilling. The height, or the depth of the oil, will be the difference between the depth of the *porous* rock and the depth of the *non-porous* rock. The conditions mentioned above must now be taken into account. Since it is impossible to know all the conditions of an oil well, an estimate must be made of the *oil in place* in a reservoir. The formula used to estimate the available oil in a reservoir is

$$\text{oil in place} = \text{area} \times \text{net to gross} \times \text{height} \times \text{porosity} \times \text{saturation}$$

Example

A certain oil field has an area of 20 km^2 with an uncertainty of ± 20% in the area of the field. Calculate the maximum and minimum area of the field in m^2.

Solution:

$$\begin{aligned} 1 \text{ km} &= 1 \times 10^3 \text{ m} \\ \Rightarrow (1 \text{ km})^2 &= (1 \times 10^3 \text{ m})^2 \\ \Rightarrow 1 \text{ km}^2 &= 1 \times 10^6 \text{ m}^2 \\ \Rightarrow 20 \text{ km}^2 &= 20 \times 10^6 \text{ m}^2 \\ &= 2 \times 10^7 \text{ m}^2 \end{aligned}$$

To calculate the margin of error we get 20% of this number.

$$\begin{aligned} \text{Error} &= 20\% \text{ of } 2 \times 10^7 \text{ m}^2 \\ &= \frac{20 \times 2 \times 10^7}{100} \text{ m}^2 \\ &= \frac{4 \times 10^8}{1 \times 10^2} \text{ m}^2 \\ &= 4 \times 10^6 \text{ m}^2 \end{aligned}$$

We add this to the estimated area to get the maximum area.

$$\begin{aligned} \text{Maximum area} &= 2 \times 10^7 \text{ m}^2 + 4 \times 10^6 \text{ m}^2 \\ &= 20 \times 10^6 \text{ m}^2 + 4 \times 10^6 \text{ m}^2 \\ &= 24 \times 10^6 \text{ m}^2 \\ &= 2.4 \times 10^7 \text{ m}^2 \end{aligned}$$

We subtract the error from the estimated area to get the minimum area.

$$\begin{aligned} \text{Minimum area} &= 2 \times 10^7 \text{ m}^2 - 4 \times 10^6 \text{ m}^2 \\ &= 20 \times 10^6 \text{ m}^2 - 4 \times 10^6 \text{ m}^2 \\ &= 16 \times 10^6 \text{ m}^2 \\ &= 1.6 \times 10^7 \text{ m}^2 \end{aligned}$$

Challenges

CHALLENGE 1

The same oil field has a N/G (net to gross) of 0.8 ± 20%. Calculate the maximum and minimum values of the net to gross. Why are there no units given to the numbers in this challenge?

CHALLENGE 2

By drilling at different sites in this same oil field some distance apart it is estimated that the height is 50 m ± 10%. What are the maximum and minimum values of the height as estimated from this drilling?

CHALLENGE 3

This same oil field has a *porosity* of (30 ± 25) % and a saturation of (70 ± 30) %. What is the expected oil in place? What are the maximum and minimum estimates of the volume of oil in place?

Teacher Page

Solutions

CHALLENGE 1:

This is very similar to the example problem.

$$\begin{aligned} N/G &= 0.8 \pm 20\% \\ &= 0.8 \pm \frac{20 \times 0.8}{100} \\ &= 0.8 \pm 0.16 \end{aligned}$$

so the maximum value is 0.96 or 96% and the minimum value is 0.64 or 64%. There are no units because the net to gross is a pure number. It is the ratio between the volume of rock considered to be good enough to be reservoir rock and the total rock volume.

CHALLENGE 2:

The calculation is more practice with percentages. If we represent the Height (or depth) of the oil by H then:

$$\begin{aligned} H &= 50\text{ m} \pm 10\% \\ &= 50\text{ m} \pm \left(\frac{10}{100} \times 50\text{ m}\right) \\ &= 50\text{ m} \pm 5\text{ m} \end{aligned}$$

which gives a maximum height of 55 m and a minimum height of 45 m.

CHALLENGE 3:

This puts everything together. First we evaluate the expected amount of oil with no margin of error.

Expected oil in place = area X net to gross X height X porosity X saturation

$= 2 \times 10^7\ m^2 \times 0.8 \times 50\ m \times 0.3 \times 0.7$

$= 1.68 \times 10^8\ m^3$

The next thing to do is use the maximum values of all the data.

Maximum $= 2.4 \times 10^7\ m^2 \times 0.96 \times 55\ m \times 0.55 \times 1$

$= 6.97 \times 10^8\ m^3$

Now perform the same calculation with the minimum values of the data to get

Minimum $= 1.6 \times 10^7\ m^2 \times 0.64 \times 45\ m \times 0.05 \times 0.4$

$= 9.2 \times 10^6\ m^3$

Comments and Suggestions

Overview: This topic shows that even the elementary use of percentages, index notation and error calculations as studied at Junior Certificate and Ordinary Level Leaving Certificate are part of an engineer's toolkit. It helps to show that all the mathematics studied at school is used in important applications by engineers throughout the world.