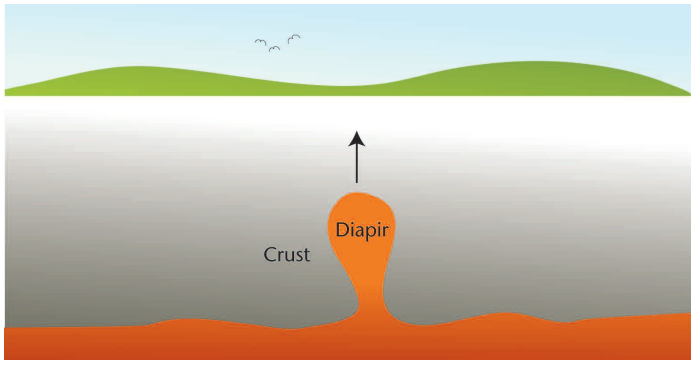
MATHEMATICAL MODELLING Series 1

Volcanic maths

Mathematics and applied mathematics are used in everyday life. Stock markets, mobile phones, car manufacturing, Google, Hollywood special effects, digital TV and satellites all use cutting-edge mathematics tools in their basic functions. The Mathematical Modelling Series presents a number of applications of mathematics in domains as varied as the human body, volcanology, telecommunications or finance.

When magma erupts from a volcano, it has already travelled a long distance from its point of origin. How the magma makes its way from deep within the Earth to the surface has been debated by geologists for decades. While there are many different theories, it is very difficult to observe what's actually happening far beneath the Earth's surface. Using mathematics we can decide which theories are physically realistic and which are not.



How it works

One theory is that the magma rises in a similar way to wax rising in a lava lamp. Rocks at the bottom of the crust are melted to form magma, which rises up in one big blob known as a diapir. The magma is less dense than the Earth's crust and so it rises by Archimedes' principle. The magma moves very slowly, taking tens of thousands of years to reach the surface. Because it is rising so slowly the solid crust has time to deform and behaves like a very thick liquid. We can calculate the temperature of the diapir by solving an equation that says that the total heat energy must be conserved. We can use this temperature, together with Newton's second law, to calculate the velocity of the diapir. This tells us whether or not the diapir will reach the surface of the Earth.

Conclusion

Solving the equations tells us that diapirs can only reach the surface of the Earth under very specific conditions. It is unlikely that most magma reaches the surface of the Earth via diapirs. This would be very difficult to determine by observation but was much easier using mathematics.

Parts of the curriculum used in this project:

- Differentiation
- Integration
- Trigonometry
- Newton's second law
- Archimedes' principle
- Ordinary differential equations

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If you want more information about MACSI and this project:

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